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LETTER TO THE EDITOR

Low-frequency response of ferronematic liquid crystal

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Abstract. The magnetomechanical response of uniformly magnetized nematic liquid crystals with positive diamagnetic anisotropy, often named ferronematics, is studied. Particular attention is given to the motions which are controlled by reactive forces originating from antisymmetric magnetic stresses. We show that in this case an incompressible ferronematic soft matter can transmit perturbation by a transverse magnetotorsion wave travelling along the applied magnetic field and that this feature of ferronematics can be utilized to measure their magnetic anisotropy. The dispersion equation characterizing the magnetotorsion wave is derived, followed by a brief discussion of the possible experimental identification of the low-frequency mode predicted.

We discuss low-frequency hydrodynamic fluctuations induced in the volume of a magnetized nematic liquid crystal with positive diamagnetic anisotropy, χ_a. Our goal is to show that this kind of nematics, often referred to as ferronematics, can transmit perturbation by transverse magnetomechanical waves exhibiting the non-Hookean rheology of nematic soft matter, and that this property of ferronematics can be adopted to measure their magnetic anisotropy.

Our considerations are based on the following experimental and theoretical observations. The magnetic field, $\boldsymbol{H}$, penetrating in the sample, intensifies the initial molecular field by the supplementary field, $\boldsymbol{h} = \chi_a(\boldsymbol{n}\cdot\boldsymbol{H})\boldsymbol{H}$, where $\chi_a > 0$ and $\boldsymbol{n}$ is the director. As shown in [1], the most prominent effect of the magnetic field is that it imparts to ferronematic matter a certain portion of magnetotorsion elasticity, which is described by antisymmetric stresses. In the presence of the field, $\boldsymbol{h}$, the total stresses in the nematic medium are described by the tensor

$$\sigma_{ik}^{\mathrm{M}} = \sigma_{ik}^{\mathrm{e}} + \tau_{ik} \qquad \tau_{ik} = \tfrac{1}{2}(n_i h_k - n_k h_i) \tag{1}$$

where σ_{ik}^{e} is the symmetric tensor of Eriksen's stresses (see equation (3.100) of section 3.5.2 in [1]) and τ_{ik} is the completely antisymmetric stress tensor introduced in [1]. The physical meaning of De Gennes–Prost's stresses, τ_{ik}, can be clarified by considering linearized equations of nematodynamics

$$\frac{\partial \delta v_k}{\partial x_k} = 0 \tag{2}$$

$$\rho\frac{\partial \delta v_i}{\partial t} = \frac{\partial \delta\sigma_{ik}^{\mathrm{M}}}{\partial x_k} \qquad \delta\sigma_{ik}^{\mathrm{M}} = \delta\sigma_{ik}^{\mathrm{e}} + \delta\tau_{ik} \qquad \delta\tau_{ik} = \tfrac{1}{2}(\delta n_i h_k - \delta n_k h_i) \tag{3}$$

$$\frac{\partial \delta n_i}{\partial t} = \epsilon_{ijk}\delta\omega_j n_k \qquad \delta\omega_j = \tfrac{1}{2}\epsilon_{jmn}\frac{\partial \delta v_n}{\partial x_m} \tag{4}$$

describing small-amplitude fluctuations of incompressible flow, provided that the molecular field is unchanged. In equations (2)–(4), ρ is the density, δv_i is the ith component of velocity and $\delta\omega_i$ stands for the ith component of vorticity (see also [1,2]). The summation over repeated indices is presumed. The conservation of energy in the process of motion is controlled by the equation

$$\frac{\partial}{\partial t}\int\frac{1}{2}\rho\delta v^2\,\mathrm{d}V=-\int\delta\sigma^{\mathrm{e}}_{ik}\delta v_{ik}\,\mathrm{d}V-\int\delta\tau_{ki}\delta\omega_{ik}\,\mathrm{d}V \tag{5}$$

where

$$\delta v_{ik}=\frac{1}{2}\left(\frac{\partial\delta v_k}{\partial x_i}+\frac{\partial\delta v_i}{\partial x_k}\right)\qquad\delta\omega_{ik}=\frac{1}{2}\left(\frac{\partial\delta v_k}{\partial x_i}-\frac{\partial\delta v_i}{\partial x_k}\right). \tag{6}$$

The link between the tensor of the rotational distortions, $\delta\omega_{ik}$, and the vector field of the vorticity, $\delta\omega_i$, is given by $\delta\omega_{ik}=\epsilon_{ijk}\delta\omega_j$. The equation of energy balance (5) is obtained after multiplication of (3) with δv_i and integrating over the volume, V, of the sample. Here, we focus solely on the bulk response of ferronematics, therefore the surface integrals in (5) have been omitted. Equations (2)–(6) exhibit the fact that fluctuations in Eriksen's stresses, $\delta\sigma^{\mathrm{e}}_{ik}$, are accompanied by symmetric strains, δv_{ik}; the behaviour typical of Newtonian viscous liquid and Hookean elastic distortions. In contrast, fluctuations in De Gennes–Prost's stresses, $\delta\tau_{ik}$, are associated with antisymmetric rotational distortions, $\delta\omega_{ik}$. In [1], it was argued that antisymmetric stresses are responsible for the transmission of torques (see section 3.5.3 of [1]). Our purpose here is to show that the torque induced by the magnetic field can be propagated in the form of transverse magnetotorsion waves.

To accentuate the key points of magnetomechanical response, we confine our consideration to dissipation-free motion of the ferronematic medium caused by perturbation of the equilibrium state in which the director, $\boldsymbol{n}$, is firmly aligned in the direction of the uniform magnetic field

$$\boldsymbol{n}=\frac{\boldsymbol{H}}{H}=\text{const}\qquad\text{and}\qquad\boldsymbol{h}=\chi_a H\boldsymbol{H}=\text{const}. \tag{7}$$

Given this, equation (4), which describes slight deflections of $\boldsymbol{n}$ from the direction of the applied magnetic field $\boldsymbol{H}$, can be represented as follows

$$\frac{\partial\delta\boldsymbol{n}(\boldsymbol{r},t)}{\partial t}=\frac{1}{H}\left[\delta\boldsymbol{\omega}(\boldsymbol{r},t)\times\boldsymbol{H}\right]. \tag{8}$$

In the incompressible nematic medium, the fluctuating velocity, $\delta\boldsymbol{v}(\boldsymbol{r},t)$, and displacement, $\delta\boldsymbol{u}(\boldsymbol{r},t)$, can be represented in the form of general rotational identities (see equation (3.108) of [1])

$$\delta\boldsymbol{v}(\boldsymbol{r},t)=\frac{\partial\delta\boldsymbol{u}(\boldsymbol{r},t)}{\partial t}=[\delta\boldsymbol{\omega}(\boldsymbol{r},t)\times\boldsymbol{r}] \tag{9}$$

$$\delta\boldsymbol{u}(\boldsymbol{r},t)=[\delta\boldsymbol{\phi}(\boldsymbol{r},t)\times\boldsymbol{r}]\qquad\delta\boldsymbol{\omega}(\boldsymbol{r},t)=\frac{\partial\delta\boldsymbol{\phi}(\boldsymbol{r},t)}{\partial t}. \tag{10}$$

This representation is dictated by the rotational character of hydrodynamic fluctuations whose development is controlled by antisymmetric stresses. Inserting this definition of $\delta\boldsymbol{\omega}(\boldsymbol{r},t)$ into (8), one finds

$$\delta\boldsymbol{n}(\boldsymbol{r},t)=[\delta\boldsymbol{\phi}(\boldsymbol{r},t)\times\boldsymbol{n}]. \tag{11}$$

Equations (9)–(11) show that the centres of inertia and director for every macroscopically small element undergo fluctuations driven by vortical flow. The above simultaneous representation of fluctuating variables has been suggested in [1] in the context of the hydrostatic balance of torques. Guided by similar motivations, we show that such a form for both $\delta\boldsymbol{n}$ and $\delta\boldsymbol{v}$ can be used efficiently to search for oscillatory behaviour of torque. For incompressible flow we can write

$$\nabla \cdot \delta \boldsymbol{v}(\boldsymbol{r}, t) = 0 \qquad \text{and} \qquad \nabla \cdot \delta \boldsymbol{u}(\boldsymbol{r}, t) = 0. \tag{12}$$

By inserting the above definition of $\delta \boldsymbol{u}$, equation (10), into (12) we immediately find: $\mathrm{div}\delta \boldsymbol{u} = \boldsymbol{r} \cdot \mathrm{curl}\delta \boldsymbol{\phi} = 0$. This equation would be an identity, if

$$\nabla \times \delta \boldsymbol{\omega}(\boldsymbol{r}, t) = 0 \qquad \text{and} \qquad \nabla \times \delta \boldsymbol{\phi}(\boldsymbol{r}, t) = 0. \tag{13}$$

Acting by operator div on $\delta \boldsymbol{n}$, we get

$$\nabla \cdot \delta \boldsymbol{n}(\boldsymbol{r}, t) = 0. \tag{14}$$

Equations (12)–(14) express kinematic constraints on the permissible motions and show that perturbed fluctuations are not accompanied by splay strains. However, these equations say nothing about possible coupling between $\delta \boldsymbol{n}$ and $\delta \boldsymbol{v}$. The main objective of our further analysis is to find the hydrodynamic mechanism providing coherent oscillations of these quantities. We show that the reactive force, $g_i = \nabla_k \delta \tau_{ik}$, can provide such a behaviour. As a result, we arrive at the following closed set of equations

$$\nabla \cdot \delta \boldsymbol{v} = 0 \qquad \nabla \cdot \delta \boldsymbol{n} = 0 \tag{15}$$

$$\rho \frac{\partial \delta \boldsymbol{v}}{\partial t} = \frac{\chi_a H}{2} \nabla \times [\delta \boldsymbol{n} \times \boldsymbol{H}] \tag{16}$$

$$\frac{\partial \delta \boldsymbol{n}(\boldsymbol{r}, t)}{\partial t} = \frac{1}{2H} \big[[\nabla \times \delta \boldsymbol{v}(\boldsymbol{r}, t)] \times \boldsymbol{H} \big]. \tag{17}$$

Equations (16) and (17) exhibit an essentially elastodynamic character of interaction between fluctuating director, $\delta \boldsymbol{n}$, and the applied magnetic field, $\boldsymbol{H}$. This interaction is mediated by rotational distortions described by the vorticity, $\delta \boldsymbol{\omega}$, of the nematodynamic flow.

Let us consider perturbation in the plane-wave form

$$\delta \boldsymbol{v} = \tilde{\boldsymbol{v}} \exp(\mathrm{i}\boldsymbol{k}\boldsymbol{r} - \mathrm{i}\omega t) \qquad \delta \boldsymbol{n} = \tilde{\boldsymbol{n}} \exp(\mathrm{i}\boldsymbol{k}\boldsymbol{r} - \mathrm{i}\omega t). \tag{18}$$

Here $\boldsymbol{k}$ stands for the wave vector and ω is the frequency of oscillations; $\tilde{\boldsymbol{v}}$ and $\tilde{\boldsymbol{n}}$ are small constant vectors. Substituting (18) into (15), one has

$$(\boldsymbol{k} \cdot \delta \boldsymbol{v}) = 0 \qquad (\boldsymbol{k} \cdot \delta \boldsymbol{n}) = 0. \tag{19}$$

Substitution of (18) into (16) yields

$$\omega \rho \delta \boldsymbol{v} = -\frac{\chi_a H}{2} (\boldsymbol{k} \cdot \boldsymbol{H}) \delta \boldsymbol{n}. \tag{20}$$

Inserting (18) into (17), we obtain

$$\omega \delta \boldsymbol{n} = -\frac{1}{2H} \left[(\boldsymbol{k} \cdot \boldsymbol{H}) \delta \boldsymbol{v} - \boldsymbol{k}(\delta \boldsymbol{v} \cdot \boldsymbol{H}) \right]. \tag{21}$$

Taking the scalar product of (21) with $\boldsymbol{k} \neq 0$ and considering (19), we get

$$(\delta \boldsymbol{v} \cdot \boldsymbol{H}) = 0. \tag{22}$$

Thus, the kinematics of simultaneous fluctuations in the velocity and director selects only those coupled oscillations for which $\delta \boldsymbol{v} \perp \boldsymbol{H}$ and $\delta \boldsymbol{n} \perp \boldsymbol{H}$. Finally, the characteristic system of equations reads

$$\omega \rho \delta \boldsymbol{v} + \frac{\chi_a H}{2} (\boldsymbol{k} \cdot \boldsymbol{H}) \delta \boldsymbol{n} = 0 \tag{23}$$

$$\omega \delta \boldsymbol{n} + \frac{1}{2H} (\boldsymbol{k} \cdot \boldsymbol{H}) \delta \boldsymbol{v} = 0. \tag{24}$$

The inspection of admissible directions of the wave propagation compatible with requirements (19) and (22), leads to the conclusion that $\boldsymbol{k}$ must be either parallel or antiparallel to $\boldsymbol{H}$: $(\boldsymbol{k}\cdot\boldsymbol{H}) = \pm kH$. With this in mind, from (23) and (24) we immediately find

$$\omega^2 = V_{\mathrm{mt}}^2 k^2 \qquad V_{\mathrm{mt}}^2 = \frac{\chi_a}{4\rho} H^2. \tag{25}$$

This dispersion equation uniquely defines the transverse magnetotorsion wave propagating with the phase velocity, V_{mt}. In this wave the director and velocity execute coupled oscillations in the plane perpendicular to $\boldsymbol{H}$. Notice that for nematics with negative diamagnetic anisotropy it must be a relaxation mode. Such a sensitive response of magnetotorsion to the sign of χ_a might be useful in the practical determination of magnetic anisotropy of synthesized new nematics. The propagation of a magnetotorsion wave in two opposite directions is the dynamical manifestation of the fact that in a magnetized ferronematic liquid crystal, the two directions $\boldsymbol{n}$ and $-\boldsymbol{n}$ are energetically equivalent. The above consideration shows that the magnetization of a randomly fluctuating nematic medium will stabilize a two-dimensional incompressible flow circulating back-and-forth in the plane, perpendicular to the applied magnetic field. From equations (23) and (24) it follows

$$\frac{\rho\delta\boldsymbol{v}^2}{2} = \frac{\chi_a H^2}{2}\delta\boldsymbol{n}^2. \tag{26}$$

That is, the mean energy of the magnetorsion wave in the kinetic motions and in the motions of director is the same. Taking numerical values $\chi_a = 1.2\times10^{-7}$, $H = 10^4$–10^5 G and $\rho = 1$ g cm^{-3} (typical of the PAA and MBBA nematics), we obtain $V_{\mathrm{mt}} \sim 1$–10 cm s^{-1}. For comparison, the speed of a longitudinal sound wave $c_s \sim 10^4$–10^5 cm s^{-1}. At $k = 10^4$ cm^{-1}, the above dispersion equation leads to the resonance frequency of the order of $\omega \sim 10^4$ s^{-1}. The resonance frequency is shifted when the intensity of the applied magnetic field is changed. The latter effect can be detected by optical means. We believe that the above physical features and estimates can be efficiently utilized as a guideline in search of the magnetotorsion mode predicted on currently operating facilities [3–5].

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